

A comparison of digital signaling schemes :

(Pp. 226 - 229)

How to compare various mod. schemes :

Compare based on SNR required to achieve certain prob. of error

- Need to put some constraint
- fixed date rate of T_b
- fixed BW

Two major classes of signaling schemes:

- (1) BW efficient signaling
- (2) Power efficient signaling

Criterion for power efficiency:

SNR per bit required to achieve a certain prob.

of error (usually schemes are compared at $P_e = 10^{-5}$)

i.e. $N_b = \frac{E_b}{N_0}$ required to achieve $P_e = 10^{-5}$

Lower $N_b \Rightarrow$ higher efficiency

Criterion for BW efficiency:

BW efficiency is measured by spectral bit rate

$$r = \frac{R}{W} \quad \leftarrow \text{bit rate} \quad (\text{b/s}) \text{ Hz}$$

Larger $r \Rightarrow$ higher BW efficiency

Good system is one which provides

lowest N_b at fixed r

highest r

From the expressions derived, we can determine the required N_b to achieve certain prob. of error (e.g. $P_e = 10^{-5}$)

(1)

Bandwidth & Dimensionality:

The only signal which is both time-limited & BW-limited is the trivial signal

All other signals have either infinite BW or infinite time duration

That said, all practical signals are approximately

time - & bandwidth limited

Now remember that by Parseval's theorem

$$Ex = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

time limited signals, limited

Now let's focus on time limited signals, so that

$$E_{T_1, T_2} = \int_{T_1}^{T_2} |x^2(t)| dt = \int_{T_1}^{T_2} |X(f)|^2 df$$

$$Ex = \int_{-\infty}^{\infty} |x^2(t)| dt$$

We can now state the dimensionality theorem:

Dimensionality theorem: Assume that $X(f)$ is η -bandwidth limited to W , i.e. at most a fraction η of the energy of $x(t)$ is outside the band $[-W, W]$,

$$\frac{1}{2\pi} \int_{-W}^W |X(f)|^2 df \geq 1 - \eta$$

Consider the set of all signals $x(t)$ with support $[-T/2, T/2]$ that are η -bandwidth limited to W . Then there exists a set of N orthonormal signals $\phi_j(t)$ ($1 \leq j \leq N$) such that $x(t)$ can be approximated by this set of orthonormal signals, i.e.

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \left(x(t) - \sum_{j=1}^N \langle x(t), \phi_j(t) \rangle \phi_j(t) \right)^2 dt \leq E$$

where $E = 12\eta^2 N T + 1$

From dimensionality theorem, we can see that

$$N \leq 2WT$$

is the dimension of space of functions limited to duration T & bandlimited to W .

Check next page for an alternative way to understand $N \leq 2WT$ obtained from sampling theorem.

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$$N = 2WT_s$$

$$R_s = \frac{W}{T_s}$$

$$N = \frac{RN}{2}$$

Also we have

$$R = R_s \log_2 M$$

so the spectral efficiency is given by

$$r = \frac{R}{W}$$

$r = \frac{2 \log_2 M}{N}$

Consider a signaling scheme with

- M signals
- With duration T_s (symbol rate)
- Appx BW. W

The dimensionality of signal space is

(3)

Maximum info rate:

2 pieces of info per second per Hz π

Assuming no noise, a channel of $BW = B$ Hz can transmit a signal of $BW = B$ Hz error free

But signal of B can be reconstructed from

its Nyquist samples which are at a rate $2B$ Hz

i.e. a signal of B can be completely specified by $2B$ indep. pieces of info per second

$$g(t) = \sum g(kT_s) \sin[\omega t - k\pi]$$

$$= \sum g(kT_s) \sin[\omega Bt - k\pi]$$

i.e. the $2B$ pieces of info per second as the values of the Nyquist samples $g(kT_s)$

channel can send this info error free \Rightarrow
2B independent pieces of info can be transmitted
per sec. & no more

Demonstrates a scheme which allows error free transmission of $2B$ indep. pieces of info per sec. per ch. $B \neq BW \cdot B$ (4)

- PAM (With SSBB Modulation)

$$N=1 \Rightarrow r = 2 \log_2 M \geq 2$$

- PAM

$$N=2 \Rightarrow r = \log_2 M \geq 1$$

QAM & PSK

$$N=2 \Rightarrow r = \log_2 M \geq 1$$

spectral efficiency

$$r = \frac{2 \log_2 M}{N}$$

As M increases, BW efficiencies decrease v. BW i.e. for large M , system becomes inefficient.

- * So in PAM, QAM, & PSK power efficiency decreases as M increases.
- * However, for these schemes, power efficiency decreases as M increases.

So size of constellation (M) determines the tradeoff between the two efficiencies.

Such systems are appropriate when BW is limited. However, increasing M improves the power efficiency. In fact, system is capable of achieving the Shannon limit as M increases.

Appropriate for power-limited channels that have sufficiently large BW to accommodate a large no. of signals.

e.g. deep space communication.

ex of such channels are

- Telephone channels
- Digital microwave channels.

(5)

These three modulation classes are one way to have a tradeoff between bandwidth & power efficiency.

Another method to do so is coding (gives practical methods to have a tradeoff bet. BW & power efficiency).

When we study the capacity of bandlimited channels, we will show that the following should hold as $P_c \rightarrow 0$, namely

$$\frac{C_b}{N_0} > \frac{2^r - 1}{r}$$

This is the condition under which reliable communication is possible which holds for any communication system.
(infinite BW)

As r tends to zero, we obtain the limit on $\frac{C_b}{N_0}$ for reliable communication

$$\frac{C_b}{N_0} \rightarrow -1.6 \text{ dB}$$